I. - TRIGONOMETRÍA

FÓRMULAS BÁSICAS

$\operatorname{sen}\alpha = \dfrac{c}{a}$

$\operatorname{cosec}\alpha = \dfrac{1}{\operatorname{sen}\alpha} = \dfrac{a}{c}$

$\cos\alpha = \dfrac{b}{a}$

$\sec\alpha = \dfrac{1}{\cos\alpha} = \dfrac{a}{b}$

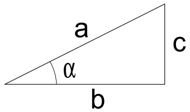
$\tan\alpha = \dfrac{\operatorname{sen}\alpha}{\cos\alpha} = \dfrac{c}{b}$

$\operatorname{cotan}\alpha = \dfrac{1}{\tan\alpha} = \dfrac{b}{c}$

$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$

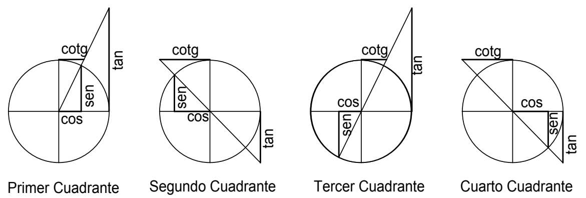
$\tan\alpha \times \operatorname{cotan}\alpha = 1$

$1 + \tan^2\alpha = \dfrac{1}{\cos^2\alpha} = \sec^2\alpha$



$1 + \operatorname{cotan}^2\alpha = \dfrac{1}{\operatorname{sen}^2\alpha} = \operatorname{cosec}^2\alpha$

LINEAS TRIGONOMÉTRICAS



REDUCCION AL 1er CUADRANTE

Ángulos complementarios:
Su suma vale π/2 radianes (90°)

sen (π/2 – α) = cos α
cos (π/2 – α) = sen α
tan (π/2 – α) = ctg α

Ángulos suplementarios:
Su suma vale π radianes (180°)

sen (π – α) = sen α
cos (π – α) = – cos α
tan (π – α) = – tan α

Ángulos que difieren en π/2 radianes:

sen (π/2 + α) = cos α
cos (π/2 + α) = – sen α
tan (π/2 + α) = – ctg α

Ángulos que se diferencian π radianes:

sen (π + α) = – sen α
cos (π + α) = – cos α
tan (π + α) = tan α

Ángulos opuestos:

sen (– α) = – sen(α)
cos (– α) = cos α
tan (– α) = – tan α

DETERMINACION DE UNA RAZON EN FUNCION DE OTRA

En función del seno:

$\operatorname{cosec}\alpha = \dfrac{1}{\operatorname{sen}\alpha}$

$\cos\alpha = \sqrt{1-\operatorname{sen}^2\alpha}$

$\sec\alpha = \dfrac{1}{\sqrt{1-\operatorname{sen}^2\alpha}}$

$\tan\alpha = \dfrac{\operatorname{sen}\alpha}{\sqrt{1-\operatorname{sen}^2\alpha}}$

$\operatorname{ctg}\alpha = \dfrac{\sqrt{1-\operatorname{sen}^2\alpha}}{\operatorname{sen}\alpha}$

En función del coseno:

$\operatorname{sen}\alpha = \sqrt{1-\cos^2\alpha}$

$\sec\alpha = \dfrac{1}{\cos\alpha}$

$\operatorname{cosec}\alpha = \dfrac{1}{\sqrt{1-\cos^2\alpha}}$

$\tan\alpha = \dfrac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}$

$\operatorname{ctg}\alpha = \dfrac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$

En función de la tangente:

$\cos\alpha = \dfrac{1}{\sqrt{1+\tan^2\alpha}}$

$\operatorname{ctg}\alpha = \dfrac{1}{\tan\alpha}$

$\sec\alpha = \sqrt{1+\tan^2\alpha}$

$\operatorname{cosec}\alpha = \dfrac{\sqrt{1+\tan^2\alpha}}{\tan\alpha}$

$\operatorname{sen}\alpha = \dfrac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}$

En función de la tangente del ángulo mitad

(Usadas para integrar)

$\operatorname{sen}\alpha = \dfrac{2\tan(\alpha/2)}{1+\tan^2(\alpha/2)}$

$\operatorname{sen} 2\alpha = \dfrac{2\tan\alpha}{1+\tan^2\alpha}$

$\cos\alpha = \dfrac{1-\tan^2(\alpha/2)}{1+\tan^2(\alpha/2)}$

$\cos 2\alpha = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$

$\tan\alpha = \dfrac{2\tan(\alpha/2)}{1-\tan^2(\alpha/2)}$

$\tan 2\alpha = \dfrac{2\tan^2\alpha}{1-\tan^2\alpha}$

En función del coseno del ángulo doble:

(Usadas para integrar)

$\operatorname{sen}\alpha = \sqrt{\dfrac{1-\cos 2\alpha}{2}}$

$\operatorname{sen}\dfrac{\alpha}{2} = \sqrt{\dfrac{1-\cos\alpha}{2}}$

$\cos\alpha = \sqrt{\dfrac{1+\cos 2\alpha}{2}}$

$\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{1+\cos\alpha}{2}}$

$\tan\alpha = \sqrt{\dfrac{1-\cos 2\alpha}{1+\cos 2\alpha}}$

$\tan\dfrac{\alpha}{2} = \sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

RAZONES DEL ÁNGULO SUMA/DIFERENCIA

$\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha\cos\beta \pm \cos\alpha\operatorname{sen}\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \operatorname{sen}\alpha\operatorname{sen}\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\dfrac{\operatorname{sen}\alpha + \operatorname{sen}\beta}{\operatorname{sen}\alpha - \operatorname{sen}\beta} = \dfrac{\tan\frac{1}{2}(\alpha+\beta)}{\tan\frac{1}{2}(\alpha-\beta)}$

$\operatorname{ctg}(\alpha \pm \beta) = \dfrac{\operatorname{ctg}\alpha\operatorname{ctg}\beta \mp 1}{\operatorname{ctg}\alpha \pm \operatorname{ctg}\beta}$

$\dfrac{\cos s\alpha + \cos\beta}{\cos\alpha - \cos\beta} = -\dfrac{\alpha+\beta}{2}\operatorname{cotan}\dfrac{\alpha-\beta}{2}$

TRANSFORMACION DE SUMAS A PRODUCTOS Y VICEVERSA

(Estas expresiones se utilizan en la resolución de triángulos con el empleo de logaritmos)

SUMAS a PRODUCTOS

$\operatorname{sen}\alpha + \operatorname{sen}\beta = 2\operatorname{sen}\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\operatorname{sen}\alpha - \operatorname{sen}\beta = 2\cos\dfrac{\alpha+\beta}{2}\operatorname{sen}\dfrac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2\operatorname{sen}\dfrac{\alpha+\beta}{2}\operatorname{sen}\dfrac{\alpha-\beta}{2}$

$\tan\alpha + \tan\beta = \dfrac{\operatorname{sen}(\alpha+\beta)}{\cos\alpha\cos\beta}$

$\tan\alpha - \tan\beta = \dfrac{\operatorname{sen}(\alpha+\beta)}{\cos\alpha\cos\beta}$

$\operatorname{ctg}\alpha + \operatorname{ctg}\beta = \dfrac{\operatorname{sen}(\alpha+\beta)}{\operatorname{sen}\alpha\operatorname{sen}\beta}$

$\operatorname{ctg}\alpha - \operatorname{ctg}\beta = \dfrac{\operatorname{sen}(\beta-\alpha)}{\operatorname{sen}\alpha\operatorname{sen}\beta}$

PRODUCTOS a SUMAS

$\operatorname{sen}\alpha\operatorname{sen}\beta = \dfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$

$\operatorname{sen}\alpha\cos\beta = \dfrac{1}{2}\left[\operatorname{sen}(\alpha+\beta) + \operatorname{sen}(\alpha-\beta)\right]$

$\cos\alpha\cos\beta = \dfrac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$

FUNCIONES DE LOS MÚLTIPLOS DE UN ÁNGULO

Ángulo doble

$\operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$

$\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

Ángulo triple

$\operatorname{sen} 3\alpha = 3\operatorname{sen}\alpha - 4\operatorname{sen}^3\alpha$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

$\tan 3\alpha = \dfrac{3\tan\alpha}{1-3\tan^2\alpha}$

FUNCIONES TRIGONOMÉTRICAS INVERSAS

$$\operatorname{arcsen} x = \arccos\sqrt{1-x^2} = \frac{\pi}{2} - \arccos x$$

$$\arccos x = \operatorname{arcsen}\sqrt{1-x^2} = \frac{\pi}{2} - \operatorname{arcsen} x$$

$$\arctan x = \operatorname{arcsen}\frac{x}{\sqrt{1+x^2}} = \frac{\pi}{2} - \operatorname{arctg} x$$

$$\operatorname{arcsen} x + \operatorname{arcsen} y = \operatorname{arcsen}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$$

$$\operatorname{arcsen} x - \operatorname{arcsen} y = \operatorname{arcsen}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$$

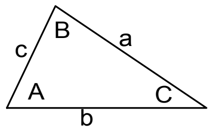
$$\arccos x + \arccos y = \arccos\left[xy - \sqrt{(1-x^2)(1-y^2)}\right]$$

$$\arccos x - \arccos y = \arccos\left[xy + \sqrt{(1-x^2)(1-y^2)}\right]$$

$$\arctan x + \arctan y = \frac{x+y}{1-xy} \qquad \arctan x - \arctan y = \frac{x-y}{1+xy}$$

FÓRMULAS DE BRIGGS

Para las tangentes de los ángulos mitad, se dividen las expresiones análogas miembro a miembro. Para el ángulo entero se utilizan las fórmulas que dan
las razones de un ángulo en función del coseno del ángulo doble. Estas
mulas ya se han tratado anteriormente.



$$\operatorname{sen}\frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}}$$

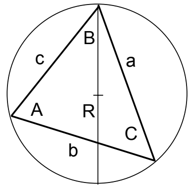
$$\operatorname{sen}\frac{B}{2} = \sqrt{\frac{(p-a)(p-c)}{ac}} \qquad \cos\frac{B}{2} = \sqrt{\frac{p(p-b)}{ac}}$$

$$\operatorname{sen}\frac{C}{2} = \sqrt{\frac{(p-b)(p-a)}{ab}} \qquad \cos\frac{C}{2} = \sqrt{\frac{p(p-c)}{ab}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{p(p-c)}{bc}} \qquad p = \frac{a+b+c}{2}$$

TEOREMAS IMPORTANTES:

Teorema de los senos:



$$\frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} = \frac{c}{\operatorname{sen} C} = 2R$$

Teorema de los cosenos:

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

Teorema de las tangentes

$$\frac{a+b}{a-b} = \frac{\operatorname{sen} A + \operatorname{sen} B}{\operatorname{sen} A - \operatorname{sen} B} = \frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$$

AREA DEL TRIÁNGULO

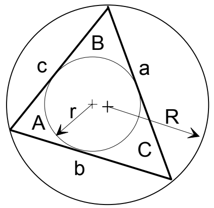
$$S = \frac{1}{2}ab\operatorname{sen} C = \frac{1}{2}cb\operatorname{sen} A = \frac{1}{2}ac\operatorname{sen} B$$

(Fórmula de Herón)

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

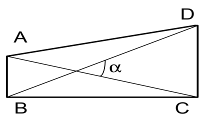
$$S = pr = \frac{abc}{4R} = \frac{p}{2R}$$

$$p = \frac{a+b+c}{2}$$



AREA DE UN CUADRILÁTERO

$$S = \frac{\overline{AC}\,\overline{BD}}{2}\operatorname{sen}\alpha$$



II.- FUNCIONES HIPERBÓLICAS

FÓRMULAS BÁSICAS

$$\operatorname{Sh}\alpha = \frac{e^{\alpha}-e^{-\alpha}}{2} \qquad \operatorname{Th}\alpha = \frac{\operatorname{Sh}\alpha}{\operatorname{Ch}\alpha} = \frac{e^{\alpha}-e^{-\alpha}}{e^{\alpha}+e^{-\alpha}}$$

$$\operatorname{Ch}\alpha = \frac{e^{\alpha}+e^{-\alpha}}{2} \qquad \operatorname{Ch}\alpha - \operatorname{Sh}\alpha = e^{-\alpha}$$

$$\operatorname{Sh}\alpha + \operatorname{Ch}\alpha = e^{\alpha} \qquad \operatorname{Ch}^2\alpha - \operatorname{Sh}^2\alpha = 1$$

FUNCIONES DEL ÁNGULO SUMA/DIFERENCIA

$$\operatorname{Sh}(\alpha+\beta) = \operatorname{Sh}\alpha\operatorname{Ch}\beta + \operatorname{Sh}\beta\operatorname{Ch}\alpha \qquad \operatorname{Sh}(\alpha-\beta) = \operatorname{Sh}\alpha\operatorname{Ch}\beta - \operatorname{Sh}\beta\operatorname{Ch}\alpha$$

$$\operatorname{Ch}(\alpha+\beta) = \operatorname{Ch}\alpha\operatorname{Ch}\beta + \operatorname{Sh}\beta\operatorname{Sh}\alpha \qquad \operatorname{Ch}(\alpha-\beta) = \operatorname{Ch}\alpha\operatorname{Ch}\beta - \operatorname{Sh}\alpha\operatorname{Sh}\beta$$

$$\operatorname{Th}(\alpha+\beta) = \frac{\operatorname{Th}\alpha + \operatorname{Th}\beta}{1+\operatorname{Th}\alpha\operatorname{Th}\beta} \qquad \operatorname{Th}(\alpha-\beta) = \frac{\operatorname{Th}\alpha - \operatorname{Th}\beta}{1-\operatorname{Th}\alpha\operatorname{Th}\beta}$$

FUNCIONES DEL ÁNGULO DOBLE/MITAD

$$\operatorname{Sh}2\alpha = 2\operatorname{Sh}\alpha\operatorname{Ch}\alpha \qquad \operatorname{Ch}2\alpha = \operatorname{Sh}^2\alpha + \operatorname{Ch}^2\alpha \qquad \operatorname{Th}2\alpha = \frac{2\operatorname{Sh}\alpha\operatorname{Ch}\alpha}{\operatorname{Sh}^2\alpha + \operatorname{Ch}^2\alpha}$$

$$\operatorname{Sh}\frac{\alpha}{2} = \sqrt{\frac{1}{2}(\operatorname{Ch}\alpha - 1)} \qquad \operatorname{Ch}\frac{\alpha}{2} = \sqrt{\frac{1}{2}(\operatorname{Ch}\alpha + 1)} \qquad \operatorname{Th}\frac{\alpha}{2} = \sqrt{\frac{\operatorname{Ch}\alpha - 1}{\operatorname{Ch}\alpha + 1}}$$

TRANSFORMACION DE PRODUCTOS A SUMAS

$$\operatorname{Sh}\alpha\operatorname{Sh}\beta = \frac{1}{2}\left[\operatorname{Ch}(\alpha+\beta) - \operatorname{Ch}(\alpha-\beta)\right] \qquad \operatorname{Ch}\alpha\operatorname{Ch}\beta = \frac{1}{2}\left[\operatorname{Ch}(\alpha+\beta) + \operatorname{Ch}(\alpha-\beta)\right]$$

$$\operatorname{Sh}\alpha\operatorname{Ch}\beta = \frac{1}{2}\left[\operatorname{Sh}(\alpha+\beta) + \operatorname{Sh}(\alpha-\beta)\right] \qquad (\operatorname{Ch}\alpha \pm \operatorname{Sh}\alpha)^n = \operatorname{Ch} n\alpha \pm \operatorname{Sh} n\alpha$$

FUNCIONES HIPERBÓLICAS INVERSAS

$$\operatorname{ArgSh} x = \ln\left(x + \sqrt{x^2+1}\right) \qquad \operatorname{ArgCh} x = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\operatorname{ArgTh} x = \frac{1}{2}\ln\frac{1+x}{1-x} \qquad \operatorname{ArgSh} x = \operatorname{ArgCh}\sqrt{x^2+1} = \operatorname{ArgTh}\frac{x}{\sqrt{x^2+1}}$$

$$\operatorname{ArgCth} x = \frac{1}{2}\ln\frac{x+1}{x-1} \qquad \operatorname{ArgCh} x = \operatorname{ArgSh}\sqrt{x^2-1} = \operatorname{ArgTh}\frac{\sqrt{x^2-1}}{x}$$

$$\operatorname{ArgTh} x = \operatorname{ArgSh}\frac{x}{\sqrt{1-x^2}} = \operatorname{ArgCh}\frac{x}{\sqrt{1-x^2}} = \operatorname{ArgCth}\frac{1}{x}$$

RELACIONES ENTRE FUNCIONES CIRCULARES E HIPERBÓLICAS

$$\operatorname{sen} x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right) \qquad \cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

$\operatorname{Sh} ix = i\operatorname{sen} x$ $\qquad$ $\operatorname{sen} ix = i\operatorname{Sh} x$ $\qquad$ $e^{ix} = \cos x + i\operatorname{sen} x$

$\operatorname{Ch} ix = \cos x$ $\qquad$ $\cos ix = \operatorname{Ch} x$ $\qquad$ $e^{-ix} = \cos x - i\operatorname{sen} x$

$\operatorname{Th} ix = i\tan x$ $\qquad$ $\tan ix = i\operatorname{Th} x$ $\qquad$ $\operatorname{arcsen} ix = i\operatorname{Arg\,Sh} x$

$\operatorname{sen}(x+iy) = \operatorname{sen} x\operatorname{Ch} y + i\cos x\operatorname{Sh} y$ $\qquad$ $\arccos ix = -i\operatorname{Arg\,Ch} x$

$\cos(x+iy) = \cos x\operatorname{Ch} y - i\operatorname{sen} x\operatorname{Sh} y$ $\qquad$ $\arctan ix = i\operatorname{Arg\,Th} x = \dfrac{1}{2}\,i\ln\dfrac{1+x}{1-x}$